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ROBUST STATISTICS FOR SPATIAL ANALYSIS:  
THE CENTER OF ACTIVITY

BY

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The center of activity (Hayne, 1949), the arithmetic mean of Cartesian coordinate vectors, has been widely used as the single best statistical estimator of an individual animal's location over a given time interval. Although several authors have questioned the biological relevance of the center of activity (Hayne, 1949; Harrison, 1958; Smith et al., 1973; Stickel, 1954; Tanaka, 1963), most workers have found it useful in itself (Barbehenn, 1974; Cooper, 1978; Doebel and McGinnis, 1974; Gipson and Sealander, 1972; Koepl et al., 1979; Post, 1974) and as a basis for many statistical home range models (Calhoun and Casby, 1958; Currie and Bellis, 1969; Dice and Clark, 1953; Harrison, 1958; Jennrich and Turner, 1969; Koepl et al., 1975, 1977; Mazurkiewicz, 1969, 1971; White, 1964).

In recent years statisticians have criticized the arithmetic mean for its sensitivity to outliers (Andrews et al., 1972; Huber, 1972). One of the virtues of the arithmetic mean is that it incorporates all observations, equally weighted, but this also is one of its weaknesses because a large error in any measurement is reflected in the sample mean; for this reason the arithmetic mean lacks robustness. Consequently, statisticians have proposed a number of robust location estimators. An excellent comparative study of 68 of these robust location estimators was performed by Andrews et al. (1972).

Because the concept of robust estimates of location may be new to vertebrate biologists, we introduce here a simple, hypothetical example of

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a robust estimate of location applied to a frequency distribution contaminated with outliers (Fig. 1). It is clear that there is a central distribution in the interval 1-9 containing 95% of the observations. Two of the observations (at 16 and 19) appear to be remote from the central distribution and are potential outliers. The mean of all observations is 5.575. When we apply a simple robust estimate of location to this sample by symmetrically trimming 5% ( $2\frac{1}{2}\%$  from each tail of the distribution) and computing the mean on the remaining observations, we achieve the value 5.461. If we increase the percentage of trim to 10 and then 20%, we see that the estimates of location which result (5.342 and 5.139, respectively) more closely approximate the mean of the central distribution (4.947). It is interesting that trimming, when performed on observations which are part of the central distribution, but ostensibly not outliers, produces only a relatively small deviation in the estimate of location. Thus, light symmetrical trimming prior to the computation of the mean seems to provide an intuitively better estimate of location than the traditional mean on the full data set. Hence this procedure qualifies as a robust estimate of location.

Another intuitive way of appreciating robust estimates of location is through the sensitivity curves discussed by Andrews et al. (1972). Suppose we had a sample of 19 normally distributed observations to which a 20th observation is added. If the 20th observation coincides with the mean of the 19 observations the mean on the full 20 observations is unaffected. However, if the 20th observation deviates from the mean of the 19 observations, the mean of the 20 observations also deviates  $\frac{1}{20}$ th of the magnitude of the deviation of the 20th observation. By varying the value of the 20th observation, while keeping the other 19 constant, and computing the estimate of location each time, we can summarize our results in a plot of the estimate of location as a function of the variable observation (Fig. 2). We call this kind of plot a sensitivity curve. The sensitivity curve of the mean is represented as a straight diagonal line because the estimate of

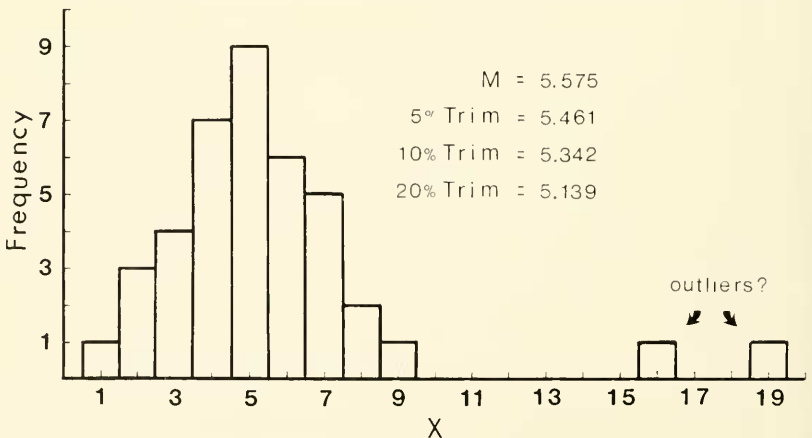


FIG. 1. A simple, hypothetical distribution contaminated by outlier (Arrows).

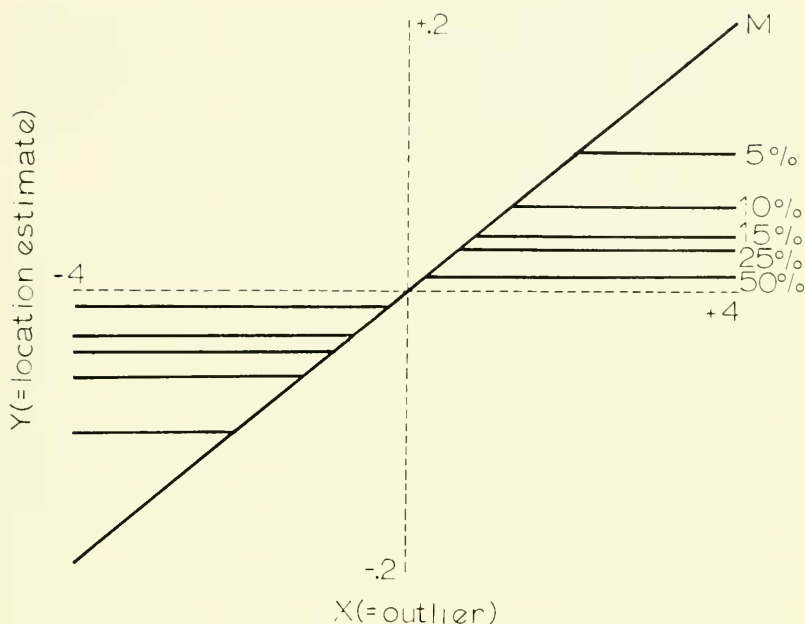


FIG. 2. Sensitivity curves for the mean (diagonal line, M) and simple robust estimates of location achieved through various percentages of symmetrical trimming of the tails of the distribution prior to computing the mean of the remaining observations.

location is always directly proportional to the deviation of the 20th point from the mean of the first 19. If we repeat this procedure using various percentages of symmetrical trim, the additional lines of Figure 2 are accounted for. It is clear from these lines that the magnitude of an estimate of location using trimmed data is bounded. When the deviation of the 20th point from the mean is small, the estimate of location is close to the untrimmed mean. As the 20th observation deviates farther from the mean, it falls into the region to be trimmed and so does not affect the trimmed estimate. More severe trims reduce the range of values which will be included in the estimate of location, thus narrowing the distance between bounds. These bounded curves are characteristic of robust estimates of location.

Because the center of activity employs the arithmetic mean, it too is sensitive to outliers, and therefore is not robust. Hayne (1949) surmised this when he wrote, ". . . the implication inherent in the algebra [for computing the center of activity] that relative importance is according to the first power of the distance and number of points of capture is entirely unproven." It is timely to appraise the traditional computation of the center of activity in light of these alternative methods and arguments. We will not comprehensively compare many of the various robust location estimators proposed because Andrews et al. (1972) have already done so.

We will, however, apply several robust estimators which Andrews et al. (1972) thought promising to two-dimensional radio-tracking data of an urban striped skunk (*Mephitis mephitis*), primarily to illustrate their properties and use, but also as an example of a practical problem frequently arising in studies of animal movement.

## MATERIALS AND METHODS

### Radio-tracking

A striped skunk captured in Lawrence, Kansas, on 6 September 1977 was fitted with a 23-gram, pulsing transmitter having a mercury controlled activity switch and whip antenna (obtained from Wildlife Materials Inc., Carbondale, Illinois), and released. Transmitter broadcasting radius was 2.1 miles and battery life  $250 \pm 50$  days. A portable 12-channel continuous frequency receiver (from the same company), with a range of 150.9 to 151.0 MHz was used to monitor the signals. Each time the skunk was located, the peak and two nulls were recorded as compass azimuths as well as time of day, location of observer, and temperature. Gain settings and signal intensity measured with a VU meter allowed estimation of receiver-transmitter distance. From these data, location coordinates of the skunk were obtained.

Three observers alternated tracking sessions. Two sessions, four hours in length, were held every third night from 7 September to 15 November 1977, totaling 48 sessions. Each session ordinarily commenced at 1900 hours and ended at 0300 hours CST. Rain during a scheduled tracking night postponed the session until the next clement evening.

## COMPUTATIONAL PROCEDURES

Computer algorithms for calculating the various estimates of location in this study were taken from Andrews et al. (1972) and adapted for FORTRAN Y; these and original algorithms for calculating the sensitivity curves and surfaces were run on the Honeywell 66/60 computer at the University of Kansas. To check location estimator algorithms we reproduced the sensitivity curves in Andrews et al. (1972). Besides the traditional mean ( $M$ ), five additional estimates have been chosen, based on their performance over a range of sample sizes and distributions. Below, we briefly describe the methods employed, but for the exact methods, refer to the algorithms supplied in Andrews et al. (1972).

### Simple Trimmed Means ( $M$ and 10%)

If the proportion ( $\alpha$ ) of observations trimmed from each end of an ordered array is a multiple of  $1/n$  ( $n$ =sample size), then an integral number of points should be deleted from each end of the sorted vector ( $v$ ). Otherwise, a weighted mean,

$$L^* = \left\{ P_{v_{([\alpha n + 1])}} + v_{([\alpha n + 2])} + \dots + P_{v_{(n - [\alpha n])}} \right\} / \{n(1 - 2\alpha)\}$$

is used, where  $P = 1 + [\alpha n] - \alpha n$ , the subscript of  $v$  denotes a specific element,  $L^*$  is the estimate of location which results, and  $[\ ]$  denotes the integral portion of quantity enclosed (see Andrews et al., 1972:7). An  $\alpha$  value of 0.0 yields  $M$ ; when  $\alpha = 0.05$  the location estimate is a 10% trim.

Restricted Adaptive Trimmed Mean (JBT)

The value for  $\alpha$  is chosen to minimize the asymptotic variance  $\hat{A}$ :

$$\hat{A}_{(\alpha)} = \frac{1}{(1 - 2\alpha)^2} \left\{ \sum_{j = \alpha n + 1}^{n - \alpha n} (v_{(j)} - L_{\alpha}^*)^2 + \alpha (v_{(\alpha n + 1)} - L_{\alpha}^*)^2 + \alpha (v_{(n - \alpha n)} - L_{\alpha}^*)^2 \right\}$$

in which  $\alpha = ([n/12]/n)$  and  $([n/4]/n)$ ; the trimmed mean having the smaller of these two variances represents the robust estimate of location (see Andrews et al., 1972:9).

M-Estimates (Sine Function, AMT; Independent Scale Piecewise, 17A)

Both M- estimates tested involved the solution to the equation

$$\Sigma \Psi \left( \frac{v - L^*}{s_1} \right) = 0 .$$

For AMT,  $\Psi(v) = \sin (v/2.1)$  where  $|v| < 2.1\pi$ . Otherwise,  $\Psi(v) = 0.0$ . The estimate of scale,  $s_1$ , used is the median of the absolute deviation about the estimate of location,  $L^*$ ; this estimate is revised every third iteration (see Andrews et al., 1972:15).

For estimate 17A, the equation above was solved for  $\Psi(v) = \text{sign of } v$  times  $y$ , where

$$y = \begin{array}{ll} |v| & 0 \leq |v| < 1.7 \\ 1.7 & 1.7 \leq |v| < 3.4 \\ \frac{8.5 - |v|}{3} & 3.4 \leq |v| < 8.5 \\ 0 & |v| \geq 8.5 \end{array}$$

where  $s_1$  is the median of absolute deviations from the median. (see Andrews et al., 1972:14).

Multiply—skipped mean, Max (5k,2 deleted) (5T1)

Hinge estimates  $h_1$  and  $h_2$  (Andrews, et al., 1972:18) are first computed on  $v$  where

$$h_1 = \text{or } \begin{array}{l} (y_{\binom{n}{4}}), n \text{ not a multiple of } 4 \\ (y_{\binom{n}{4}} + y_{\binom{n}{4} + 1})/2, n \text{ a multiple of } 4 \end{array}$$

$$h_2 = \begin{cases} v_{(n+1 - \lfloor \frac{n+3}{4} \rfloor)}, & n \text{ not a multiple of } 4 \\ \text{or} \\ (v_{(n+1 - \frac{n}{4})} + v_{(n - \frac{n}{4})})/2, & n \text{ a multiple of } 4. \end{cases}$$

Then,  $t_1$  and  $t_2$ , scale estimates furthest from the center of the data are computed as:

$$\begin{aligned} t_1 &= h_1 - 1.5 (h_2 - h_1) \\ \text{and } t_2 &= h_2 + 1.5 (h_2 - h_1) \end{aligned}$$

An initial skipping procedure deletes or skips observations lying outside the scale estimates ( $t_1$  and  $t_2$ ). If  $k \geq 1$  observations are deleted by the above procedures a further  $\max(2k, 1)$  are deleted from the end of the array and the mean is computed for the observations remaining (Andrews et al., 1972:18-20).

#### Sensitivity Surfaces

To study the behavior of bivariate outliers on bivariate sample distributions using the six estimates of location described above, we plotted sensitivity surfaces, the three-dimensional extension of the sensitivity curves (Andrews et al., 1972). The method was as follows: the  $x$  and  $y$  sample coordinate vectors for 19 points were scaled to the interval  $-1.0$  to  $+1.0$  so that the sample estimate of location was at the origin  $(0.0, 0.0)$ , and the relative position in space of the sample coordinates was thus preserved. A known outlier value was added to these scaled and centered sample data. The  $x$ - and  $y$ - coordinate values of the outlier were then systematically and independently varied from  $-5.0$  to  $+5.0$  at intervals of  $0.100000$  and  $0.166667$  for the  $x$ - and  $y$ -axes, respectively. Deviations (distances) between the sample activity center and the location estimate for samples with known outliers were represented by the height ( $z$ -coordinate) for any  $x$ - and  $y$ -outlier combination. The height above the surface has been shown by 51 different symbols in increasing intervals of  $0.01$ . Every second interval is represented by a different symbol so that the final figure resembles a contour map and can be similarly interpreted. In all, each sensitivity surface is represented by 5150 discrete printed symbols; sensitivity surface facsimiles in the present paper (Fig. 3) have been drawn from these computer generated contour maps. A similar procedure was followed using the 10 unique locations for the radio-collared skunk, contaminated with a single outlier (Fig. 4). Note that frequency of occurrence at each location was disregarded.

#### Relative Importance Index

To determine the effect of each sample observation on the location estimators we computed a relative importance index. This entailed computing the Euclidean distance between the activity center of the entire

sample ( $n$ ) and the activity center with an observation deleted ( $n-1$ ), standardized by the distance between the activity center and the location coordinates of the deleted observation. We then multiplied by  $(n-1)$  to account for the fact that observations from small samples would be expected to have more influence than those from larger samples. In

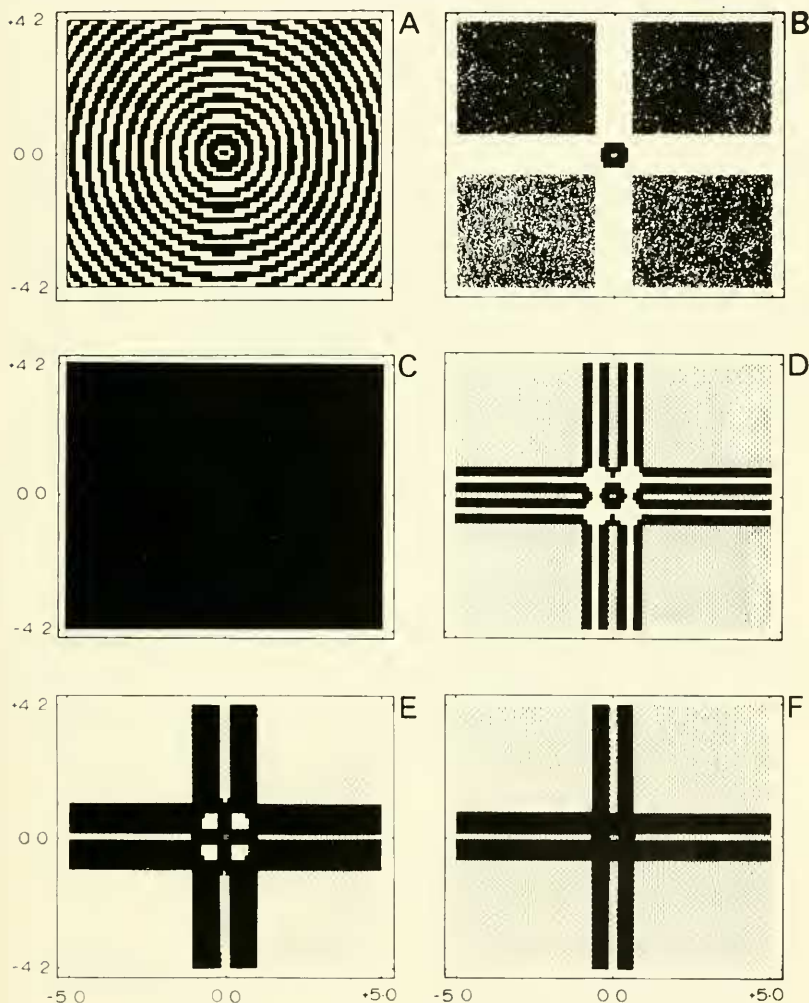


FIG. 3. Sensitivity surfaces for six algorithms of the center of activity on idealized bivariate normal data to which a variable outlier was adjoined: A. arithmetic mean(M); B. 10 percent trim (10%); C. restricted adaptive trimmed mean(JBT); D. M-estimate, sine function weighted (AMT); E. M-estimate, independent scale piecewise weights(17A); F. multiply skipped mean, max 5K, 2 deleted(5T1). Each sensitivity surface can be interpreted like a topographic map, but with the different shading representing different surface levels. For A, levels are shown as alternating black and white bands with the lowest values at the center, radiating outward. The remaining surfaces (B-F) in increasing magnitude, are shown by dashed lines, black, white, and gray.

mathematical terms, the relative importance index for the  $i$ -th observation ( $\omega_i$ ) may be represented as:

$$\omega_i = \frac{[(L_{x_n}^* - L_{x_{(n-1)}}^*)^2 + (L_{y_n}^* - L_{y_{(n-1)}}^*)^2]^{1/2}(n-1)}{[(L_{x_n}^* - L_{x_1}^*)^2 + (L_{y_n}^* - L_{y_1}^*)^2]^{1/2}}$$

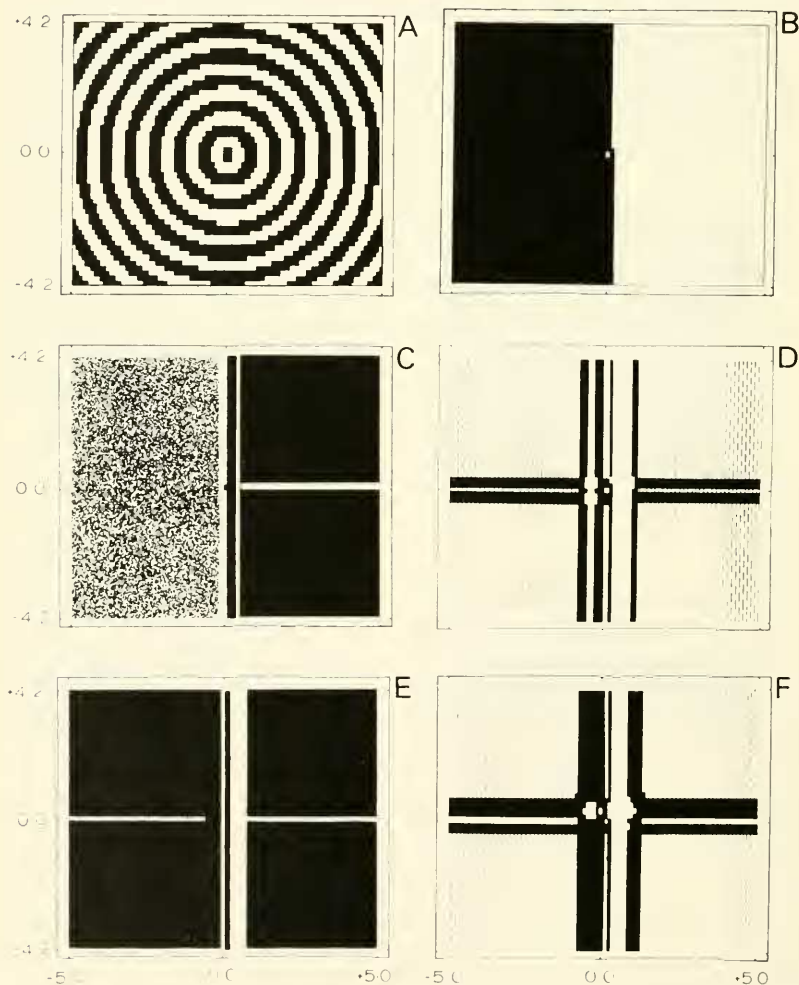


FIG. 4. Sensitivity surfaces computed for six algorithms of the center of activity on the 10 unique coordinates of the skunk data to which a variable outlier was adjoined. See legend of Figure 3 for further details.



When  $L_{x_n}^*$  is the estimate of location calculated for all  $n$ ,  $x$  coordinates;

$L_{y_n}^*$  is the estimate of location calculated for all  $n$ ,  $y$  coordinates;

$L_{x_{(n-1)}}^*$  is the estimate of location of  $n-1$ ,  $x$  coordinates;

$L_{y_{(n-1)}}^*$  is the estimate of location of  $n-1$ ,  $y$  coordinates;

$L_{x_i}$  is the  $x$   $i$ -th coordinate of the abscissa;

$L_{y_i}$  is the  $y$   $i$ -th coordinate of the ordinate.

The importance index may vary from 0 to  $\infty$ ; by definition  $\omega_i = \infty$  when the divisor of the above equation is zero. A  $\omega_i$  value near zero denotes a coordinate whose deletion does not influence the location estimator. Values of  $\omega_i = 1.0$  result from the arithmetic mean.

Several robust estimates of location use weighting factors which are similar in concept to our relative importance index. However, the importance index can be calculated for any proposed estimate, not just those employing weighting factors, thereby facilitating comparison.

## RESULTS

### Sensitivity Surfaces

The bivariate sensitivity surface (Fig. 3A) clearly shows that the activity center or bivariate mean is unbounded when an idealized symmetrical distribution is contaminated by outliers of increasing magnitude. This is indicated by the regular pattern of the concentric contours, which in theory form an inverted cone with its vertex at the plot center.

The remaining sensitivity surfaces for the robust estimates (Fig. 3B-F) show different patterns; all achieve plateaus at the margins of the plots and we can assume that they remain at the same or lower level if extended infinitely. Another notable difference between the surface representing the bivariate mean and those representing robust estimators is the existence in the latter of valleys radiating from the center of the surface, oriented parallel to the fixed axes of the surfaces. The valleys are due to independent consideration of the  $x$  and  $y$  coordinates, in which outliers are recognized by extreme  $x$  or  $y$  coordinates but not by intermediate values of both. A better technique for identifying outliers would be to rank observations according to distance from the center of activity, perhaps using a standard distance such as a Mahalanobis distance which would adjust for elongated home ranges.

When the 10 skunk locations are used, the location and shape of the cone representing the traditional activity center are the same (Fig. 4A). It is interesting that although the sample data are not symmetrical, the

sensitivity surface is. This is another indication that the traditional activity center is invariant to rotation of the data. Symmetry, and by inference, invariance to rotation, is not exhibited by the sensitivity surfaces for the robust estimates of location (Fig. 4B-F). However, their robustness is still apparent from the plateau features which extend to the figures' margins.

#### Analysis of the Full Data Set

Seventy-nine location fixes were obtained for the skunk (Fig. 5). Of these, 90% occurred at two den sites. One den site was under a rear porch (Fig. 5, location 3), and the other was under a small shed (Fig. 5, location 4). Remaining points represent the location of capture (Fig. 5, location 1), an isolated observation (Fig. 5, location 2), and a single night's foray (Fig. 5, broken line, locations 5-10). Although the actual location fixes varied about the den sites, we attributed the variation to observational error, amounting to approximately  $\pm 0.25$  grid units.

Estimates of location were computed for all 79 observations and the relative importance indices for each point were determined (Table 1). The indices reveal that locations 1 and 2 (Fig. 5) are less important than the others, which make up a cluster of points around the den sites. Each of the robust estimates of location is nearer the principal distribution of data points than is the traditional activity center.

When the same statistics are computed on 10 unique skunk locations which are less heavily centralized (Table 1), the effect on the relative weights and estimates of location were similar but less pronounced.

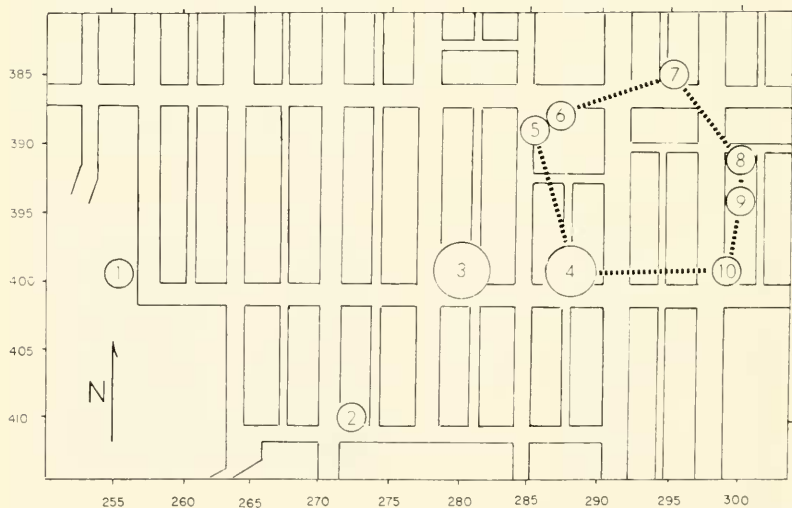


FIG. 5. Seventy-nine location fixes of an urban striped skunk (*Mephitis mephitis*) encompassed in 10 unique localities (1-10). The broken line represents the route traveled on a single night's foray.

TABLE 1. Relative importance indices and estimates of the center of activity center compared.

obs.	Full Data Set										Abridged Data Set					
	x	y	n	M	10%	JBT	AMT	17A	5T1	n	M	10%	JBT	AMT	17A	5T1
1	256.0	399.5	1	1.000	0.213	1.563	0.013	0.011	0.183	1	1.000	0.626	1.268	0.746	0.656	0.667
2	272.0	410.0	1	1.000	0.377	2.646	0.010	0.028	0.340	1	1.000	0.851	1.900	1.021	1.135	1.653
3	278.0	398.0	30	1.000	0.875	5.797	0.024	0.019	0.905	1	1.000	0.981	5.210	0.921	0.919	3.880
4	282.0	399.5	41	1.000	1.920	11.863	0.192	1.116	1.260	1	1.000	1.229	5.644	1.536	1.652	1.286
5	288.0	398.0	41	1.000	1.281	20.799	0.956	0.657	1.912	1	1.000	1.229	5.644	1.536	1.652	1.286
6	288.5	400.0	1	1.000	1.364	27.438	1.595	1.707	1.265	1	1.000	1.210	5.043	1.090	1.208	1.286
7	285.0	389.0	1	1.000	0.559	5.195	0.019	0.076	0.133	1	1.000	1.210	5.043	1.090	1.208	1.286
8	287.0	388.0	1	1.000	0.519	4.953	0.119	0.153	0.206	1	1.000	1.328	4.059	1.194	1.340	1.286
9	295.0	385.0	1	1.000	0.402	3.420	0.013	0.035	0.234	1	1.000	1.220	1.411	1.257	1.322	1.035
10	300.0	391.0	1	1.000	0.410	3.548	0.014	0.039	0.330	1	1.000	1.463	0.908	1.087	0.752	0.556
Total	300.0	394.0	1	1.000	0.425	3.857	0.016	0.080	0.356	1	1.000	1.472	0.702	1.076	0.792	0.410
L*	299.0	399.0	1	1.000	0.292	4.432	0.024	0.018	0.282	10	1.000	1.445	0.847	1.063	0.700	0.610
L*			79	285.3	285.5	286.1	288.3	288.3	285.4		286.2	288.3	286.2	287.2	287.8	288.3
L*				398.7	399.1	399.1	399.2	399.2	399.2		395.5	394.9	395.5	395.1	395.0	395.0

## DISCUSSION

Computation of an activity center is often useful in summarizing the locational data of an individual as a single point. The method of Hayne (1949) based on independent arithmetic means has been the traditional method for accomplishing this task but it lacks robustness because it is unduly sensitive to outliers or bad data. In the past, researchers have dealt with this problem by (1) subjectively discarding the apparent outliers, or (2) using special knowledge of the data as a guide in deciding which observations were valid and which were spurious. In our example, the capture point was likely to be an outlier because skunks frequently shift their home ranges after capture and release (Verts, 1967; Verts and Storm, 1966). (3) Some researchers partition their data set into several arbitrary subsets for separate analysis. Unfortunately, this only relegates the outlier to a smaller subset; the problem remains. (4) Purists argue that to practice any of the above options introduces arbitrary biases, so the activity center should be computed on all the data, in the hope of obtaining cancelling error. (5) All of the above are rather *ad hoc* treatments of outliers in spatial data. More systematic operational procedures are widely available (Gnanadesikan and Kettenring, 1972; Grubbs, 1969; Brown, 1975; Gentleman and Wilk, 1975; Rohlf, 1975). The researcher should make a careful concerted effort to identify outliers and distributional peculiarities in the data set prior to analysis. Beyond the obvious benefits of identifying outliers, the researcher also gains familiarity with the data. Outliers in themselves are not "bad," but are often records of interesting but transitory phenomena. By definition they tend to obscure the measurement of central tendency or main effects. The researcher can devise techniques to identify and propagate outliers for closer study, or can eschew them when determining measures of spatial central tendency.

Robust estimates of location offer a reasonably good estimate of the center of activity regardless of the presence of real or suspected outliers. In exchange, some of the sensitivity of the location estimate is necessarily lost.

Robust estimates of location can enhance spatial analysis in a number of ways. First, it is difficult to identify all outliers in the large amount of data which can be collected methods such as radio telemetry. Second, spatial data of wildlife often fail to follow identifiable statistical distributions; robust estimates of location are less dependent on distributional assumptions of the arithmetic mean. Third, wildlife make forays which often produce outliers. And fourth, wildlife frequently shift their home ranges, while researchers are unable to determine when and where the shift actually occurred (Cooper, 1978).

All of the results discussed thus far indicate that the robust estimates of location differ less among themselves than they do from the arithmetic mean. Hence, the choice of a robust estimate to use, if it is necessary, is almost arbitrary. We favor using a simple trim method because it is intuitive, effective, and easy to compute.

In conclusion, robust estimates of the activity center provide reliable

estimates of location even with outliers in the data, but reduce sensitivity. If outliers in spatial data are suspected, robust estimates of the center of activity are a prudent alternative to traditional analysis, because of their high sensitivity to outliers. Using the mean to compute the center of activity may be the worst of the available alternatives.

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#### SUMMARY

The center of activity is useful as an estimate of an individual's location in space and as a basis for many statistical home range models, but it is sensitive to outliers (locational data remote from the principal distribution). We tested some newly developed "robust" estimates of location, which are less sensitive to outliers as alternatives. We have illustrated their properties by means of sensitivity surfaces and relative importance indices, and applied these robust estimates to locational data obtained by radio-tracking an urban striped skunk (*Mephitis mephitis*).

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